

# A Critique of Revised Basel II

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## Abstract

This paper critiques the revised Basel II capital requirements for banks. To provide a framework for analysis, the XYZ theory of regulatory capital is formulated. Independent of the XYZ theory, we argue that the revised Basel II capital rule for credit risk is not a good approximation to the ideal rule. Based on this, and using the XYZ theory, we argue that: (1) the revised Basel II rules should not replace the existing approaches for determining minimal capital standards, but should be used in conjunction with them, and (2) that calibrating the capital rules to maintain aggregate market capital is a prudent procedure.

## 1 Introduction

In March 2006 the Federal Reserve Board issued a notice of proposed regulation (Basel II NPR [21]) that documents the revised/new set of capital requirements that the U.S. regulatory agencies will introduce based on the June 2004, revised Basel II framework [2]. These proposed regulations are scheduled for implementation no earlier than 2008 ([21], p. 101). The June 2004 revised Basel II framework is a modified version of the July 1988 Basel I report issued by the Bank for International Settlements, Basel Committee on Banking Supervision (BCBS).

This paper provides a critique of the Basel II NPR [21] and the revised Basel II [2] capital adequacy rules. To provide a framework for analysis and discussion, we first introduce a dynamic theory for regulatory capital, called the XYZ theory of regulatory capital. After introducing the XYZ theory, we then apply it to the revised Basel II framework. Our conclusions are multiple fold.

The first conclusion is independent of the XYZ theory of regulatory capital.

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1. The revised Basel II capital rule - the Internal Ratings Based IRB (foundation or advanced) approach - is a very rough approximation to an ideal capital rule, and as such, leaves open much room for further improvement. The next two conclusions follow from the XYZ theory of regulatory capital.
2. Due to the roughness of the approximation, the revised Basel II capital rule should only be used in conjunction with other existing approaches for determining minimal capital standards. Revised Basel II should not replace the alternative approaches. This is consistent with the Basel II NPR proposed joint satisfaction of the revised Basel II rules and the FDI-CIA rule (see [21], pp. 73 and 86.). In addition, the inclusion of a parallel run and transitional floor periods as described in the Basel II NPR ([21], pp. 96-100) is a prudent procedure consistent with the implications of the XYZ theory.
3. Scaling the required capital in individual banks in order to maintain aggregate industry capital is a valid restriction for better approximating the ideal level of regulatory capital. Similarly, requiring that the regulations be modified if a 10% reduction of aggregate capital results after implementation (see [21], p. 84) is a sound trigger, consistent with the implications of the XYZ theory.

An outline for this paper is as follows. The next section introduces the XYZ dynamic theory of regulatory capital. Section 3 applies this theory to the revised Basel II proposals. Section 4 analyses the rule for determining capital in revised Basel II, and section 5 concludes.

## 2 The XYZ Theory of Regulatory Capital

This section formulates the XYZ dynamic theory of regulatory capital. The purpose of this theory is to provide a framework for understanding and evaluating the revised Basel II proposals.

### 2.1 Setup

We are given a filtered probability space  $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \in [0, \infty)}, P)$  describing the randomness in the economy where  $P$  is the statistical probability measure. Let  $\Lambda_t$  be a  $\mathbb{R}^d$ -valued vector process representing the characteristics of the banks in the economy (sometimes a bank is indexed by a superscript  $i$ ) and the characteristics of the macro-economy (the business cycle characteristics). This is called the state variables vector. We assume that  $\mathcal{F}_t$  is the  $\sigma$ -algebra generated by the state variables vector process  $\Lambda_t$ .

### 2.2 The Bank's Optimal Capital

We let  $X_t = f(t, \Lambda_t)$  be a  $\mathbb{R}$ -valued adapted stochastic process representing the bank's optimal time  $t$  capital, determined independently of any regulatory

rules.<sup>1</sup> This functional representation is almost without loss of generality, it follows if  $X_t$  is adapted and a Markov process.  $X_t$  is the level of capital that maximizes shareholder's wealth. It does not include externalities that are not internal to the bank.<sup>2</sup> The risk of any bank depends on time  $t$  and the state variables of the system. As the state variables change (both at the bank level and macro level), the optimal capital should change.

Banks may or may not know  $f(\cdot, \cdot)$ . It depends on the level of sophistication of the bank's internal risk management operation. It would certainly be true for the large and internationally active banks required to adopt the IRB approach in the U.S., see Basel II NPR [21], pp. 89-91. Currently, 11 banking organizations meet this criteria.<sup>3</sup> For those banks that do not know  $X_t$ , the process of computing the capital requirements in revised Basel II may better enable them to quantify  $X_t$ . Capital refers to tier 1, 2 and 3, as defined in the revised Basel II framework [2].

### 2.3 The Ideal Regulatory Capital

We let  $Z_t = h(t, \Lambda_t)$  be a  $\mathbb{R}$ -valued adapted stochastic process representing the ideal level of the bank's capital, as determined by the regulatory authorities *as if they had perfect knowledge*. This is the conceptual ideal. Note that the ideal  $Z_t$  is random, and can change over time as the state of the economy and the bank changes. In principle, the ideal capital level would incorporate all externalities that a bank's failure has on the economy and its liability holders, including all those costs that are not directly borne by the bank's shareholders.

This costly externalities argument is well known in the banking literature (see, for example, Kashyap and Stein [18]), and it provides the justification for why regulatory capital is necessary. The costly externalities argument motivates our first hypothesis.

**Hypothesis 1: The ideal level of regulatory capital exceeds the bank's optimal capital, i.e.**

$$Z_t > X_t.$$

Hypothesis 1 provides a lower bound on the ideal level of regulatory capital  $Z_t$ . The lower bound is the level of the bank's optimal capital.

As asserted numerous times in the revised Basel II framework [2], the regulatory authorities view the proposed capital rule as a first step and only as an approximation to the ideal capital rule. This motivates hypothesis 2.

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<sup>1</sup>Where  $f(\cdot, \cdot)$  is Borel measurable. In the subsequent subsections, it is assumed that  $g(\cdot, \cdot)$  and  $h(\cdot, \cdot)$  are also Borel measurable.

<sup>2</sup>Note that deposit insurance, if not properly priced, could induce a distortion in the bank's optimal capital, see Black, Miller and Posner [5]. There is substantial evidence that FDIC deposit insurance is priced below market levels, see Duffie, Jarrow, Purnanandam and Yang [9].

<sup>3</sup>The criteria is either (i) the bank has consolidated total assets of at least \$250 billion, or (ii) consolidated total on balance sheet foreign exposure of at least \$10 billion at the most recent year end (see [21], p. 90).

**Hypothesis 2: The ideal capital  $Z_t$  is not known to the regulators.**

Alternatively stated, the function  $h(\cdot)$ , is not known to the regulators. This is due to (i) uncertainty over the exact form of the correct risk measure to be used to determine the ideal level of regulatory capital, (ii) insufficient data for fitting the correct risk measure, and (iii) frictions in the implementation of the ideal rule (learning and implementation costs).

## 2.4 The Required Regulatory Capital

We let  $Y_t = g(t, \Lambda_t)$  be a  $\mathbb{R}$ -valued adapted stochastic process representing the bank's required time  $t$  capital, as specified by the regulatory authorities in the revised Basel II framework [2]. Regulators specify the function  $g(\cdot)$ .  $Y_t$  is the regulator's best estimate of  $Z_t$ . Of course, the goal is for regulators to set  $Y_t$  equal to  $Z_t$ . The problem is that  $Z_t$  is not known (hypothesis 2) and must be determined using economic theory (logic) and observations of lower bounds on  $Z_t$ . Lower bounds will be observed based on banking industry failure history and knowledge of the regulatory capital rules. For example, if regulatory capital is set such that only 0.1 percent of banks should fail in any year, and if the observed failure experience is statistically significantly greater than this percentage, then one can infer that the regulatory capital is set too low.

Although it is possible that the required capital  $Y_t$  is larger than that which is ideal  $Z_t$ , our last hypothesis assumes that the regulators are cautious in the imposition of capital constraints, and that they approximate  $Z_t$  from below. This is done in order that they not require too much capital, and make a bank non-competitive in the financial markets. Maintaining the competitive level of the banking industry is a key concern repeatedly expressed in the Basel II NPR document ([21], pp. 86-88).

**Hypothesis 3: The required regulatory capital is bounded above by the optimal regulatory capital**

$$Y_t < Z_t. \quad (1)$$

This hypothesis has two parts. One is that  $Y_t \neq Z_t$ , and two is that  $Y_t < Z_t$ . Hypothesis 3 needs additional justification. For the revised Basel II proposal, we will argue in the next section that  $Y_t < Z_t$  is in fact true.

Without additional structure, there is no additional relationship provable between  $X_t$  and  $Y_t$ . If one adds the hypotheses that  $X_t$  is known by the bank, and that the bank's truthfully report  $X_t$  to the regulators, then consistent with hypothesis 1, one might assert that  $X_t < Y_t$ . Unfortunately, this line of reasoning has multiple difficulties. First, as noted earlier, banks may not know the optimal capital rule  $f(\cdot, \cdot)$ . Second, due to asymmetric information and differential incentives, it can be argued that banks would not truthfully reveal the ideal capital level to regulators, in the hopes that the required capital level  $Y_t$  would be set below  $X_t$ . Third, and independent of the last two remarks, there is a view

that for many banks, the bank's optimal capital level exceeds that required by the regulators (see [22]). This view states that for competitive reasons, banks need to maintain a higher credit rating than that implied by regulatory capital levels.

The previous structure in conjunction with hypotheses 1-3 comprise the XYZ dynamic theory of regulatory capital. The theory is dynamic because  $X_t$ ,  $Y_t$  and  $Z_t$  are stochastic processes.

## 2.5 Theorems

The following theorems follow trivially from hypotheses 1 - 3.

**Theorem 1** *Given hypotheses 1 and 2. Let hypothesis 3 hold, and let  $g_j(t, \Lambda_t)$  for  $j = 1, \dots, N$  represent a collection of regulatory capital rules. Then,  $Y_t = \max[g_j(t, \Lambda_t) : \text{all } j]$  provides a (weakly) better approximation to  $Z_t$  than any single rule. If hypothesis 3 does not hold, then there is no simple ordering of the regulatory capital rules with respect to  $Z_t$ .*

In applying this theorem to the regulatory capital process, the idea is that there has been a collection of rules applied in the past, say  $g_j(t, \Lambda_t)$  for  $j = 1, \dots, N - 1$ , and now a new rule  $g_N(t, \Lambda_t)$  is being proposed. This theorem implies that we should not discard the old rules, but build upon them when introducing the new rule. The new rule implemented should not be  $g_N(t, \Lambda_t)$  but  $\max[g_j(t, \Lambda_t) : \text{all } j]$ .

Note that if hypothesis 3 does not hold, then there is no simple ordering of the regulatory rules possible. And, there is no simple method for comparing required capital to the (unknown) ideal regulatory capital.

**Theorem 2** *Given hypotheses 1-3. Let  $g^i(t, \Lambda_t)$  for  $i = 1, \dots, m$  be the existing regulatory capital for the  $i$ th bank. Then, when considering a new rule  $g_{new}^i(t, \Lambda_t)$ , the aggregate level of regulatory capital should satisfy:*

$$\sum_{i=1}^m g_{new}^i(t, \Lambda_t) \geq \sum_{i=1}^m g^i(t, \Lambda_t). \quad (2)$$

This theorem follows directly from theorem 1. It states that when adding new rules, we should insure that, in aggregate, the regulatory capital does not decrease. We point out that the hypothesis of this theorem can be weakened. Instead of hypothesis 3 holding at the individual bank level, as long as expression (1) holds at the aggregate level, this inequality will also apply.

Together, these two theorems provide us with a framework which we can use to analyze a regulatory capital determination process. The next section applies the XYZ theory to the revised Basel II proposals.

## 3 Revised Basel II

This section applies the XYZ theory of regulatory capital to revised Basel II [2]. We argue that the XYZ theory is appropriate, and draw various conclusions from this application. We do this by considering each hypothesis 1-3 in turn.

### 3.1 Hypotheses 1 and 2

Hypothesis 1 and 2 were discussed previously. As noted therein, hypothesis 1, the costly externalities argument, is well accepted in the banking literature (see, for example, Kashyap and Stein [18]), and it generates the reason regulatory capital is necessary. And, hypothesis 2, that the ideal regulatory capital rule is unknown, is confirmed by reading the revised Basel II framework which acts under the presumption that the ideal regulatory capital is only approximated by the proposed procedures.

### 3.2 Hypothesis 3

Hypothesis 3 requires some discussion. The critical part of hypothesis 3 is that the required capital is less than the ideal capital. Since the ideal capital is not observable, it is difficult to provide direct evidence to either validate or reject this hypothesis. However, there is some indirect evidence consistent with the satisfaction of this hypothesis. In the revised Basel II document, the BCBS states that all banks should have sufficient capital such that the probability of insolvency is less than .001, i.e. on average, a bank will fail only once in every thousand years (see [3], p. 11). This is a statement concerning a property of the ideal regulatory capital level.<sup>4</sup> If the ideal regulatory capital satisfies this property, and the required regulatory capital exceeded it (the contradiction of hypothesis 3), then all banks would be of the highest credit rating, estimated according to Moody's 1 year corporate default rates over 1920-2004 ([20], Exhibit 17, p. 16).<sup>5</sup> But, there are many banks that are not in this credit class. This implies, at least for these banks, that the current required level of capital is significantly less than the ideal. That is, these banks satisfy hypothesis 3.

There is another theory based argument justifying that hypothesis 3 holds for revised Basel II. As stated in Basel II [3], the credit risk adjustment for the asset values is based on the asymptotic single risk factor (asrf) model of Gordy [12]. This asrf model is utilized in order to insure that portfolio invariance holds for the capital rules.<sup>6</sup> This portfolio invariance is desired to minimize the computational costs of implementing the revised Basel II modifications. Continuing with our argument, this asrf model assumes that asset specific/idiosyncratic risks are infinitesimal and diversified away in a bank's portfolio (in the limit as the number of assets approach infinity). As such, the required capital based on the limiting case, is less than that which would be required by an ideal model taking into account the finiteness of the actual bank's portfolio. This implies the satisfaction of hypothesis 3 because the revised Basel II rule is strictly less

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<sup>4</sup>There is a difference between a risk measure satisfying this property, and the risk measure being defined by this characteristic. Value at Risk (VaR) is defined by this characteristic. The ideal capital level could satisfy this property and still not be the VaR risk measure.

<sup>5</sup>Over the years 1920-2004, the average 1 year default rates were: Aaa (0.0000), Aa (.0006), A (0.0008), Baa (0.0031), Ba (0.0139), B (0.0456), and Caa-C (0.1507).

<sup>6</sup>Portfolio invariance is required so that the required capital for a portfolio is the sum of the required capital for the component assets, i.e. the required capital is linear in the component assets' capital.

than the ideal regulatory capital level that would be required for a bank with only a finite number of assets. Of course, finite asset holdings are the actual situation and the asymptotic case is, at best, a rough approximation.

Finally, there is a third argument that also supports hypothesis 3, that the required capital is less than the ideal regulatory capital. As stated previously, there is a view (see Federal Reserve Board Governor Susan Schmidt Bies speech [22]) that for many banks, the bank's optimal capital exceeds that required by the regulators. Indeed, Governor Bies argues that this is true because a bank needs for competitive reasons to maintain a higher credit rating than that implied by regulatory capital levels.<sup>7</sup> If this belief is true, then since  $X_t < Z_t$  (by hypothesis 1 - due to costly externalities), and  $Y_t < X_t$  (via this view), then  $Y_t < Z_t$ .

### 3.3 Application of the XYZ theory

Given that hypotheses 1 - 3 can be applied to the revised Basel II document, we can interpret the conclusions of theorems 1 and 2 in this regard. These arguments are generic, independent of the new required capital rule specified. But, of course, the hope is that the new rule provides a better approximation of the ideal capital rule. This is the topic of the next section.

First, theorem 1 suggests that when implementing a new rule, that this should be done conservatively. The best rule is one that performs at least as well as all the existing rules. New rules should be implemented without discarding the old ones. If the old ones are worse approximations, then they will not be binding, and therefore obsolete. If this is not the case, then they should be used in conjunction or in parallel. This is consistent with the Basel II NPR proposed joint requirement that the bank comply with the revised Basel II rules and the FDICIA rule (see [21], p. 73 and 86.). The joint requirement of the (FDICIA) leverage based capital rule should not be removed.

In addition, the Basel II NPR ([21], p. 96-100) proposals require a four year parallel run with transitional floor periods on the magnitude of the reduction in acceptable capital. During the parallel run period, the banks adopting the IRB approach would need to compute the minimal capital levels using both the standardized and IRB approaches.<sup>8</sup> Each year, during this parallel run period, the minimal capital could be reduced by at most the transitional floor levels of 95% in year one, 90% in year 2, and 85% in year 3. The parallel run with transitional floor periods is a prudent procedure consistent with theorem 1.

Theorem 2 asserts that scaling up the new capital rule for individual banks in order that the aggregate level of capital in the banking industry does not decline (see Basel [2] p. 12) is a valid restriction. The scale factor is currently proposed to be 1.06. This scale factor was determined based on the 3rd Quantitative

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<sup>7</sup>This view does not contradict the first argument given in this section because this view only applies to those banks that are the most capitalized, and as noted previously, not all banks are in the highest credit rating.

<sup>8</sup>The leverage ratio minimal capital level must also be computed during this parallel run period and thereafter.

Impact Study completed by the BCBS in May 2003. This scale factor is tentative and potentially subject to future revision (see [21], p. 70). Similarly, requiring that the regulations be revisited/modified if a 10% reduction of aggregate capital results after implementation (see [21], p. 84) is a sound trigger, consistent with the implications of theorem 2.

## 4 The Revised Basel II Capital Rule

The quality of the approximation of the required regulatory capital is crucial to the soundness of the banking system, and consequently, the current regulatory capital rule proposed within revised Basel II needs to be critically analyzed. The conclusions of this section are independent of the XYZ theory of regulatory capital presented earlier.

To keep the argument simple, we focus only on the required tier 1 capital, although as will become obvious, the same arguments apply to the more complex rules that include both tier 2 and tier 3 capital. In revised Basel II [2], p. 12, considering only tier 1 capital, we have that the required tier 1 capital can be represented by the rule<sup>9</sup>:

$$g(t, \Lambda_t) = 0.08 \times (\sum_j \text{risk weighted asset values}_j) \times \alpha$$

where  $j$  denotes the  $j$ th asset held in the bank's asset portfolio, the risk weighted asset values are those specified within the revised Basel II framework (discussed below), and  $\alpha$  is a scale factor applied for assets with credit risk, currently set at 1.06. Note that the scale factor adjustment is a valid procedure according to theorem 2 above.

The risk weightings are explicitly adjusted for credit risk, operational risk, and market risk (see [2], p. 6, the chart). Liquidity risk is implicit in the credit risk computation and explicitly included in the market risk calculations. We now discuss the risk weighting adjustments for each of these risks.

### 4.1 Credit Risk

Credit risk is the risk of a loss due to a financial contract not paying off as promised. There are two approaches for computing the risk weighted asset values to adjust for credit risk: the standardized approach and the internal ratings based approach (foundation or advanced).

In the standardized approach, the risk weights are determined by the current credit ratings of the assets held. Tables are provided for each asset class, based on credit ratings. For the standardized approach, these credit risk weightings are discrete (lumpy) and do not correspond closely to the state variables (the dynamic nature of the asset risks). The criticism that the standardized approach is not sensitive enough to differences in credit risk is well known (e.g., see Jarrow

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<sup>9</sup>This function is only intended to be illustrative, not precise. The precise rule is given in [2].



and Turnbull [17], p. 592) and we therefore concentrate on the internal ratings based approach for our analysis.

For the internal ratings based approach (the advanced), these risk weights are based on using the bank's own estimates of the probability of default (PD), the loss given default (LGD), and the exposure at default (EAD).<sup>10</sup> These estimates are input into a formula for the determination of the capital (K) held for an asset. This capital is based on the Value at Risk (VaR) risk measure. We next discuss each of these components in more detail.

#### 4.1.1 Probability of Default (PD), Loss Given Default (LGD), Exposure at Default (EAD)

Default is defined for wholesale and retail exposures in Basel II NPR [21], pp. 109 and 111. A wholesale exposure is a credit exposure to a company, individual or government entity, and a retail exposure is a credit exposure to an individual or small business managed as part of a portfolio of similar exposures (see [21], pp. 158 and 161). A wholesale exposure defaults if either the bank determines the borrower is unlikely to pay or the borrower is at least 90 days past due on a coupon or principal payment. A retail exposure defaults if it is 120 days past due (unless it is a revolving retail exposure, then it must be 180 days past due). These definitions are consistent with the current academic literature (see Guo, Jarrow, Zeng [11]).

It is known that default probabilities and losses given default are state dependent and vary with the business cycle (see Chava and Jarrow [7]). In the revised Basel II capital requirements, LGD is not state dependent, but only quasi state dependent, since it must be computed based on an economic downturn ([2], p. 103). The same is true for the determination of the EAD ([2], p. 105). In contrast, the PD is defined to be the 1 year *long term average* default probability ([2], p. 99). This is explicitly not state dependent.<sup>11</sup>

The fact that these quantities are not state dependent implies that regulatory capital will not be state dependent. This means that the same capital will be required when the business environment over the next year is projected to be healthy as when the business environment over the next year is projected to be dismal. In principle, to maintain the same risk of bank failure across the business cycle, the required capital should be less in the first scenario, and more in the second. The current rules do not include this level of precision.

Of course, state dependence of the ideal and required capital rules is the most general approach consistent with changing business cycles and changing risk preferences in the economy. There is a concern, however, that making required capital state dependent (in particular, positively correlated to the business cycle) may exacerbate business cycles (see [18], [22]). Indeed, the concern is that when there is an economic downturn and credit is scarce, more capital will be

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<sup>10</sup>For the IRB foundation approach only the PD are provided by the bank, with the LGD and the EAD determined by formula/tables given within the Basel II framework.

<sup>11</sup>The asymptotic single risk factor model discussed below is used, in part, to make the PD state dependent. As seen below, this is at best only a partial adjustment for state dependence.

required, making it harder for banks to obtain the additional equity, and thereby increasing the level of interest rates in the economy. This concern is perhaps unwarranted.

This concern overlooks the important *offsetting and counter cyclic effect* that results precisely due to increased capital reducing the risk of bank failures during an economic downturn. Maintaining a healthy banking system during an economic downturn (avoiding bank failures, and in the extreme case, bank runs) is a certainly a counter cyclical regulatory policy. Given this insight, the concern that making required capital state dependent will exacerbate the business cycle is perhaps over stated. Furthermore, the Federal Reserve Board has at its disposal other monetary tools it can use to help reduce the level of interest rates during an economic downturn. But, maintaining the level of regulatory capital appears to be one of the few effective tools available that reduces the risk of bank failures.

#### 4.1.2 Capital K

The determination of capital for credit risk is based on Value at Risk (VaR) with a 1 year horizon with a .999 confidence level, with an embedded correlation assumption across assets (a single risk factor), and a maturity adjustment for the asset's maturity (see Basel II [3]).

**Problems with VaR as a Risk Measure** Recall that 99.9% *VaR* is defined as

$$VaR(L) = \inf\{x \geq 0 : P[L \leq x] \geq 0.999\}$$

where  $L$  is the loss on the asset portfolio. It is well known (see Gordy [12], p. 218 or Bluhm, Overbeck, Wagner [6], p. 167) that VaR has numerous conceptual problems. First, it ignores the distribution of losses beyond the target 0.999 confidence level. Second, it can penalize diversification. That is, the VaR of a diversified portfolio can be higher than for a single individual asset. Consider the following example adapted from Bluhm, Overbeck, Wagner [6], p. 168. Suppose there are two independent loan investments A and B with the loss distributions given in the following Table.

<i>Loss</i>	$P(L_A)$	$P(L_B)$	$P\left(L_{\frac{A+B}{2}}\right)^{12}$
\$0	0.9991	0.9991	0.99820081
\$0.5	0	0	0.00179838
\$1	0.0009	0.0009	0.000000081

We compare a portfolio consisting of a dollar invested in A versus a dollar invested equally in A and B. The non-diversified portfolio, A, has  $VaR(L_A) = \$0$

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<sup>12</sup>Note the because of independence:  $P\left(L_{\frac{A+B}{2}} = \$0\right) = (.9991)(.9991)$ ,  $P\left(L_{\frac{A+B}{2}} = \$.5\right) = 2(.9991)(.0009)$ ,  $P\left(L_{\frac{A+B}{2}} = \$1\right) = (.0009)(.0009)$ .

because no losses occur with probability greater than .001. But, the diversified portfolio has  $VaR\left(L_{\frac{A+B}{2}}\right) = \$50$ . Capital is needed for the diversified portfolio, but not for the single investment portfolio. Stated in more concrete terms, for the same dollar investment, a diversified loan portfolio should require less capital than for a concentrated loan portfolio. But, using VaR, we could get just the opposite capital requirement.

These two problems make VaR not the ideal risk measure to use for the determination of regulatory capital. Various alternative risk measures have been proposed which avoid these pitfalls, see Jarrow [14] for one such measure based on market prices.

**Given VaR, Problems with the Selected Implementation** There are five problems identified with the implementation of the VaR based capital charge as required in the revised Basel II framework.

**1. Portfolio Invariance.** The asymptotic single risk factor (asrf) model of Gordy [12] is imposed to guarantee portfolio invariance. Portfolio invariance is required so that the required capital for a portfolio is the sum of the required capital for the component assets, i.e. the required capital is linear in the component assets' capital. This is desired to minimize implementation costs. Unfortunately, this assumption implies that a diversified portfolio will have the same required capital as a concentrated portfolio, providing a perverse incentive towards concentrating risk.<sup>13</sup>

**2. Single Risk Factor.** As noted by Gordy [12], p. 222, a single systematic risk factor is clearly inconsistent with the evidence. For example, Duffee [8] fit a 3-factor model to corporate bond prices (2 factors for interest rates and 1 factor for default). With more than one risk factor, portfolio invariance fails implying that the required capital formula is in error.

**3. Common Correlation.** When implementing the asrf model, for non-defaulted wholesale exposures<sup>14</sup>, the revised Basel II [3], p. 64 or NPR Basel II [21], p. 185 assumes that the correlation between assets (within an asset class) is a simple function of the asset's PD. The simple function is bounded below by .12 and above by .24. At best, this is a very crude approximation to actual correlations between asset losses.

**4. Normal Distribution.** The procedure assumes that losses are normally distributed (Basel II [3], p. 5).<sup>15</sup> For assets, (percentage) losses are equal

<sup>13</sup>Concentration risk is supposed to be handled separately in the second pillar of the revised Basel II ([2], pp. 204 - 225) by supervisory discretion.

<sup>14</sup>This does not apply to high volatility commercial real estate. HVCRS uses a similar, but slightly different approximation ([21], p. 185).

<sup>15</sup>Contrary to this assertion, the Basel II NPR [21], p. 66, states that the losses are assumed to be lognormal. But, this statement in the Basel II NPR document is in error. The formula on p. 185 and the clarifying document [3] have losses entering as a dollar amount. They do not enter as the natural logarithm of a dollar amount.

to minus (percentage) returns. Thus, the assumption is equivalent to assuming that asset returns are normally distributed. This assumption is only a crude approximation. First, it is inconsistent with limited liability for asset returns (the usual assumption is lognormal returns, not normal returns). Second, it excludes stochastic volatility and jump processes for asset returns (losses). There is a wide consensus in the literature that the stochastic volatility and jump models are needed to understand asset returns (for example see Jarrow, Li and Zhao [13]).

**5. Maturity Adjustment.** The formula for the required capital imposes an adjustment for the maturity of the asset under consideration. The reasoning for this adjustment is explained in Gordy [12] pp. 210-212. Roughly, the logic is as follows. The original model pertains to the book value of the assets, and not to the market value. To adjust the model for market values, we need to adjust for an asset's maturity. The reason is that long dated assets can lose value if the issues are downgraded over the 1 year horizon, and the book value approach does not include this downgrading loss.

Unfortunately, this logic is inconsistent with current asset pricing theory (see Bielecki and Rutkowski [4]). Asset downgrades are the result of a firm's deteriorating position, and they are (in general) independent of the particular asset's (fixed income security's) maturity. In addition to the firm's health, an asset's downgrade might depend on the asset's collateral and seniority, but not its maturity (see Acharya, Bharath and Srinivasan [1], footnote 10). This is completely analogous to noting that the 1 year default probability of an fixed income security is independent of its maturity. This is because the default probability is based on the firm's assets and cash flows. The firm is the fundamental entity, not any single liability that the firm issues. It is true that some adjustment needs to be made for capital gains/losses on the assets due to market prices changing over a 1 year horizon, but the maturity adjustment is not it.

**Significance of the Approximation Error** The credit risk adjustment for the determination of capital based on the asrf VaR model has just been shown to provide only a (rough) approximation to the ideal capital rule. The question remains as to the magnitude of the approximation error.

To gauge the severity of the approximation error, one could use an alternative rule, and compute the differences in the required regulatory capital based on the alternative rule and revised Basel II. This exercise was recently performed by Kupiec [19] where the alternative rule was based on VaR in a hypothetical Black-Scholes-Merton economy where a bank's asset portfolio consists of a collection of risky zero coupon bonds following correlated geometric Brownian motions. In this setting, the alternative rule is arguably the ideal rule ( $Z_t$ ) defined in hypothesis 2 above. Here, Kupiec shows that the revised Basel II IRB advanced capital rule significantly under estimates the capital required by the alternative rule. The implication of Kupiec's results for this paper is that they justify the belief that the revised Basel II capital rules provide a poor approximation to the ideal capital rule.

## 4.2 Operational Risk

Operational risk is the risk of a loss due to inadequate or failed internal processes (people or systems) or from external events. Operational risk includes legal risk (law suit losses). Operational risk adjustments for regulatory capital can follow one of three approaches ([2], p. 144): the basic indicator approach, the standardized approach, and the advanced measurement approach. The first two are based on the bank's income flow - the required capital is set proportional to it. The third computation is left to the discretion of the bank subject to it being VaR based with a .999 confidence level and a one year horizon ([21], p. 135), of course, subject to regulatory approval and supervision.

The rules are very primitive at this stage in the revised Basel II framework. In a recent paper, Jarrow [15] categorizes operational risks of two types: system based and agency based. System based operational risks are due to the firm's operating *system*, i.e. a failure in a transaction or investment, either due to an error in the back office (or production) process or due to legal considerations. Agency based operational risks are due to *incentives*, including both fraud and mismanagement. The second type of operational risk represents an agency cost, due to the separation of a firm's ownership and management. For system type operational risks, the assumption that these risks are proportional to the bank's income flow is reasonable. However, for the agency type operational risks, these risks would not be proportional to the bank's income flow. Unfortunately, the agency type operational risks are the more important category for the determination of operational risk caused bank failures. This risk is not captured in the standardized approach, and at present, they will only be included if a bank uses the advanced measurement approach.

There is another small issue that Jarrow's [15] analysis sheds some light on. In the calculation of the required capital, unless the bank can show otherwise, capital is required to cover both expected operational risk losses plus unexpected losses (see [2], p. 151). This is in contrast to credit risk, where the VaR computation is intended to just cover unexpected losses (see [2], p. 52) and expected losses are dealt with otherwise (see [2], p. 86). Jarrow shows (in theory) that expected operational risk losses are no greater than zero, hence there should be no capital required for expected losses, but only for the unexpected losses.

## 4.3 Market Risk

Market risk is defined as the risk of loss due to movements in market prices, in particular interest rates, equities, foreign currencies and commodities. Market risk is that risk for which the required capital determination has been the most analyzed in the revised Basel II document, and that risk which is the most written about in the financial press. The VaR internal ratings based approach was introduced in the 1996 (market risk) Amendment to the 1988 Basel I procedures. For this reason, my discussion of market risk will be brief.

First, the capital adjustments for market risk are less complex (and similar conceptually) to those applied to credit risk. Consequently, the same limitations

concerning the use of VaR for credit risk apply equally well to the use of VaR for market risk. The standardized approach risk weighted values are based on a set of tables, and the internal models approach is based on a VaR approach with a .99 confidence interval and a 10 day holding period (see [2], p. 195). A scale factor of 3 is applied (analogous to the scale factor of 1.06 for credit risk) to account for model error and adverse market volatility (see [21], p. 63).

Last, the differences in the horizons, the confidence level, and the scale factors in the computation of the credit risk VaR versus the market risk VaR are problematic. Indeed, these differences could result in "regulatory arbitrage" if these differences imply different levels of capital and some credit risky investments (e.g. loans) can be categorized as falling under either the credit risk or the market risk requirements. This is an open question that needs further investigation.

## 4.4 Liquidity Risk

Liquidity risk is the risk of a loss due to the inability to sell a position at a reasonable price in a reasonable period of time. Liquidity risk is captured implicitly in the capital requirements for credit risk. In the computation of LGD, EAD it is implicitly included by requiring estimates based on an economic downturn ([2], p. 96). When computing market risk, liquidity risk is recognized in the standardized approach ([2], pp. 162 and 182) and in the internal models approach ([2], p. 200). There is current research suggesting better and more direct ways of including it into a VaR computation, see Jarrow and Protter [16].

## 5 Conclusions

The first conclusion is that the revised Basel II required capital rule (standardized or IRB (foundation or advanced)), does not generate a good approximation to the ideal regulatory capital. The major problems are that the risk measure (VaR) is not conceptually appropriate, and that the assumptions used to implement VaR are inconsistent with market evidence. The second conclusion, based on the XYZ theory of regulatory capital, is that (due to conclusion one) the revised Basel II capital rule should only be implemented in conjunction with alternative existing rules for determining capital, e.g. the FDICIA leverage based rule, and it should not replace the existing rules. Last, the third conclusion, also based on the XYZ theory of regulatory capital, is that scaling the required capital rule to maintain the aggregate banking capital is prudent.

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